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RECONSTRUCTION OF THE SPECTRAL SORPTIVITY OF A SEMITRANSPARENT MATERIAL

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A method is suggested for reconstruction of the spectral absorptivity of a semitransparent material at high temperatures. It is based on solution of the inverse radiation-conduction heat transfer problem, using experimental information on the temperature distribution at several points inside the specimen.

Most dielectrics and semiconductors are semitransparent materials, in which at high temperatures the energy is transferred simultaneously by conduction and radiation. At present, further development of methods for determination of both the thermal and optical properties of these materials is of great interest. Without knowledge of these primary properties it is impossible to calculate adequately temperature and heat flux fields, to use effectively these materials in various fields of high-temperature engineering, or to produce articles from the materials.

Known experimental methods for determination of the optical properties of semitransparent materials are based on solution of the inverse radiation transfer problem. They assume a physical statement of the problem that simplifies substantially its mathematical description or where the heat transfer equation has a simple asymptotic solution. Assumptions are usually made for the optical thickness, the radiation mode inside and on the surface of the medium, and the natural radiation of the material. Spectroscopic methods are based on photometric measurements of reflection, transmission, and emission spectra [1, 2].

In this study the spectral absorptivity is determined by numerical solution of the inverse radiation-conduction heat transfer problem. The fact that in high-temperature heating natural radiation of the material and the presence of temperature gradients inside the material affect substantially the heat transfer process is taken into account. Temperatures at several inner points of a specimen rather than spectral quantities are used as initial information.

Let us consider a layer of a selectively absorbing, radiating material under nonequilibrium conditions that are caused by a temperature difference of the radiating boundaries. The homogeneous and isotropic material (an infinite plate of finite thickness) has a molecular thermal conductivity λ and a spectral volumetric absorptivity k_ξ that are independent of temperature. The boundaries of the layer are diffusely radiating and reflecting; heat is transferred to the environment by radiation according to the Stefan-Boltzmann law.

Assuming steady-state and one-dimensional heat transfer in the layer, we write the set of dimensionless equations for the radiation-conduction heat transfer [3]:

$$\frac{d^2T}{dx^2} = \frac{1}{\mathrm{Nr}} \int_{\xi_1}^{\xi_2} Q(x, \xi')\, d\xi' , \quad 0 < x < 1 , \tag{1}$$

$$\frac{dT}{dx} = \frac{1}{4\mathrm{Nr}} \varepsilon_1^* (T^4 - T_1^4) , \quad x = 0 , \tag{2}$$

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$$\frac{dT}{dx} = \frac{1}{4\mathrm{Nr}} \varepsilon_2^* (T_2^4 - T^4), \quad x = 1. \tag{3}$$

The dimensionless spectral volumetric density of the resultant radiation $Q(x, \xi)$, characterizing the power of internal heat sources caused by the volumetric absorption of radiation, is found from the following relations [4]:

$$Q(x, \xi) = \begin{cases} \tau_\xi (I_{\xi p}(T(x) - 0.5B_\xi(x)), & 0 < x < 1, \\ 0.5\tau_\xi (I_{\xi p}(T(x) - 0.5B_\xi(x)), & x = 0 \text{ or } x = 1, \end{cases} \tag{4}$$

$$B_\xi(x) = I_\xi(T(0)) E_2(\tau_\xi x) + I_\xi(T(1)) E_2(\tau_\xi(1 - x)) \times$$

$$\times \int_0^1 \tau_\xi I_{\xi p}(T(x')) E_1(\tau_\xi |x - x'|) dx', \tag{5}$$

$$I_\xi(T(0)) = (\varepsilon_{1\xi} I_{\xi p}(T(0)) + 2\rho_{1\xi}\varepsilon_{2\xi} I_{\xi p}(T(1)) E_3(\tau_\xi) +$$

$$+ 2\rho_{1\xi} \int_0^1 \tau_\xi I_{\xi p}(T(x')) E_2(\tau_\xi x') dx' + 4\rho_{1\xi}\rho_{2\xi} E_3(\tau_\xi) \times$$

$$\times \int_0^1 \tau_\xi I_{\xi p}(T(x')) E_2(\tau_\xi(1 - x')) dx'/(1 - 4\rho_{1\xi}\rho_{2\xi} E_3^2(\tau_\xi)), \tag{6}$$

$$I_\xi(T(1)) = (\varepsilon_{2\xi} I_{\xi p}(T(1)) + 2\rho_{2\xi}\varepsilon_{1\xi} I_{\xi p}(T(0)) E_3(\tau_\xi) +$$

$$+ 2\rho_{2\xi} \int_0^1 \tau_\xi I_{\xi p}(T(x')) E_2(\tau_\xi(1 - x') dx' +$$

$$+ 4\rho_{1\xi}\rho_{2\xi} E_3(\tau_\xi) \int_0^1 \tau_\xi I_{\xi p}(T(x')) E_2(\tau_\xi x')) dx')/(1 - 4\rho_{1\xi}\rho_{2\xi} E_3(\tau_\xi^2)), \tag{7}$$

$$\tau_\xi = k_\xi D. \tag{8}$$

The inverse problem consists in finding from the additional condition

$$T(x_i) = F_i, \quad i = \overline{1, L}, \quad 0 \le x_1 < x_2 < \ldots < x_L \le 1, \quad L \ge 2, \tag{9}$$

the spectral optical thickness τ_ξ, $\xi \in (\xi_1, \xi_2)$. Then, with τ_ξ known it is possible to determine the spectral absorption coefficient k_ξ from relation (8).

The proof of the uniqueness of the solution of the problem of identifying the source function in the heat conduction equation can be found in [5-7].

The problem posed is solved in the extremal formulation by nonlinear estimation of the parameters. The parameter vector of the approximation of the spectral optical thickness τ_ξ is denoted by $\bar{a} = \{a_j\}$, $j = \overline{1,N}$.

The parameter vector is evaluated from the solution of the following extremal problem:

$$\bar{\alpha} = \arg\min_{\bar{a}} \sum_{i=1}^{L} (T(\bar{a}, x_i) - F_i)^2, \tag{10}$$

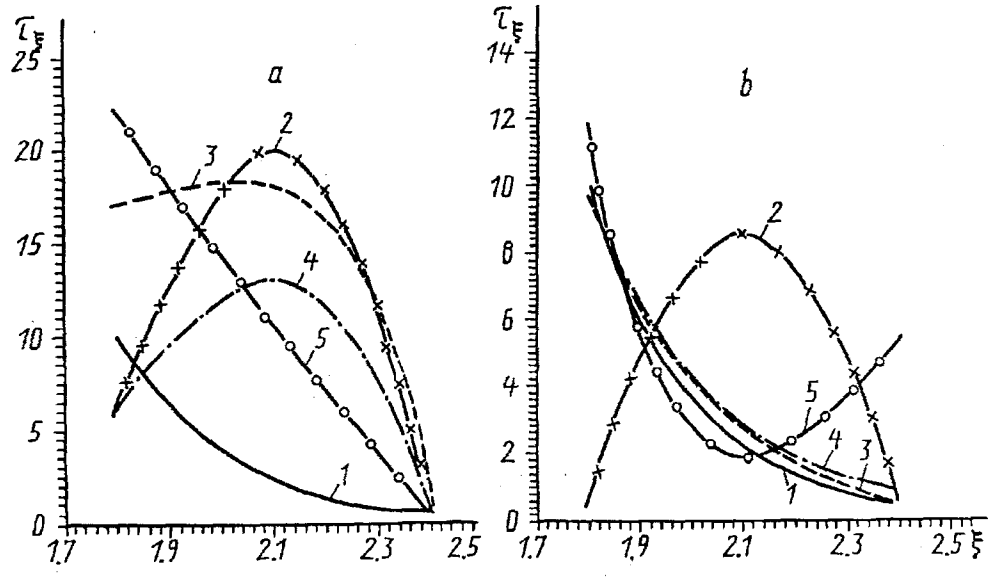

Fig. 1. Spectral optical thickness τ_ξ with the accuracy of the input data of 10^{-3} (a), 10^{-4} (b), and with different initial approximations: 1) exact values; 2) $\overset{\circ}{a} = 2$, 3) 3, 4) 3.95, 5) 5.

where $T(\bar{a}, x_i)$ are calculated temperature values obtained from the solution of the radiation-conduction heat transfer problem (1)-(7) and corresponding to the parameter vector $\bar{a}$. The solution of problem (10) is found by the conjugated gradient method with halt of the iteration process according to the iteration regularization principle [8].

In order to improve the iteration convergence and to obtain more accurate results it could be useful to carry out preliminary investigations on optimal planning of temperature measurements. In this study a locally optimal measurement plan is found from the condition of the maximum of information matrix trace [9]

$$\bar{p}_* = \arg \max_{\bar{p}} \operatorname{sp} M , \tag{11}$$

where $\bar{p} = (L, (x_i)_{i=1,L})$ is the measurement plan.

The information matrix is calculated as follows:

$$M = A^T A , \quad A = \left\{ \partial T_i / \partial a_j \right\} , \quad i = \overline{1, L} , \quad j = \overline{1, N} , \tag{12}$$

where A is the matrix of sensitivity functions of the temperature to variations of the parameter vector.

Extremal problem (11) is solved by the trial-and-error method on a set of admissible plans: $\Pi = \{\bar{p}: L \geq N, 0 \leq x_i \leq 1\}$.

The initial approximation in the form $\overset{\circ}{a}_* = \text{const}$ in the spectral range considered is chosen from condition (10).

With the normal distribution of the measurement errors, 99% of the confidence region limit for the vector of parameters $\bar{\alpha}$ satisfies the relation

$$\bar{\alpha} \pm 2.576\bar{\sigma} , \tag{13}$$

where $\bar{\sigma}$ is the vector of the variation of the estimated parameters $\bar{\alpha}$ [10].

In order to check the algorithm described above, numerical experiments were carried out for quartz glass, whose thermal and optical properties are studied most completely among the known semitransparent materials [11]. The parameters of the boundary condition $T_1 = 0.5$, $T_2 = 1$, $\varepsilon_1^* = 0.05$, $\varepsilon_2^* = 1$ and the radiation-conduction parameter Nr = 0.0517 correspond to the situation where the left-hand "cold" wall of the plane layer of the material studied

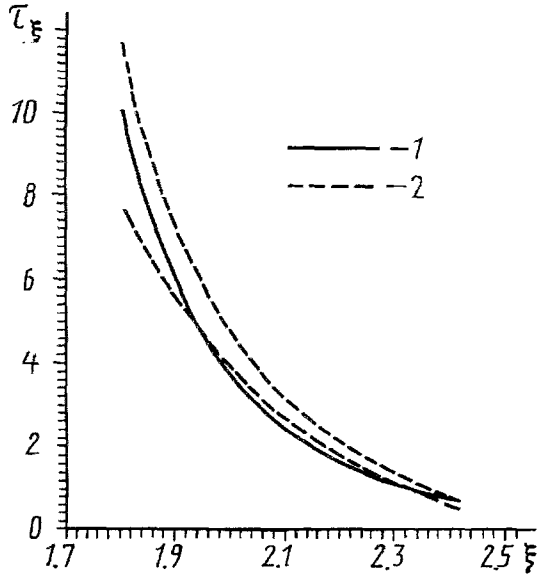

Fig. 2. Confidence region for the spectral optical density τ_ξ: 1) exact values; 2) boundaries of the confidence region.

is bounded by a nontransparent fully reflecting film and the right-hand "heated" wall is bounded by a nontransparent completely absorbing film; inside the layer the contributions of radiation and heat conduction to the total heat flux are approximately equal.

The input information on the internal temperature of the specimen F_i, $i = \overline{1, 3}$ was simulated by solution of the direct radiation-conduction heat transfer problem (1)-(7), using an implicit finite-difference scheme with a preset function τ_ξ. For simulation of measurement errors, the temperature values obtained were rounded off to a required number of decimal places. The accuracy of the input information was also limited by the approximation error of the finite-difference scheme. For the spectral absorptivity desired the piecewise-constant approximation $\tau_\xi = a_j$, $\xi \in [\xi_j, \xi_{j+1}]$, $j = \overline{1, 3}$ was used.

The numerical experiments carried out gave the following results. The optimal number and location of the temperature sensors $\bar{p}_* = \{L = 3, x_1 = 0.6, x_2 = 0.7, x_3 = 0.8\}$, obtained from the solution of problem (11), are independent of the initial approximation within a wide range of the initial values $\mathring{a}_* \in (2, 5)$. The initial approximation approximately minimizing the fuctional is $\mathring{a}_* = 3.95$.

Figure 1 shows a strong dependence of the solution on the initial approximation and on the accuracy of the input data. This can be explained by the complicated relationship between the functional to be minimized and the desired parameters as well as by ill-posed conditions of the problem. The results confirm that the proposed method for choosing the initial approximation is reasonable.

Figure 2 shows the confidence region for the solution of problem (10). The initial approximation corresponded to $\mathring{a}_*$, the measurement plan to $\bar{p}_*$, and input information was given with an accuracy of 10^{-4}. It can be seen from Fig. 2 that the desired function is located mainly in this region except for a small deviation, corresponding to the frequency interval with optical thickness values from 1 to 4. This can be explained by the fact that for the obtained temperature range $0.612 \le T \le 0.734$ the maximum values of the spectral intensity $I^{\max}_{\xi_p}(T)$ of equilibrium radiation fall within the low-frequency range $\xi < 1.86$, and therefore the results for this spectral region are more accurate.

Thus, a method is suggested for determination of the spectral absorptivity of semitransparent materials in the high-temperature range, using experimental data on the temperature distribution at several points inside the specimen. Results of numerical calculations for quartz glass showed that the algorithm proposed is efficient for high-accuracy temperature measurements and the results are reliable for the spectral range corresponding to the maximum of the spectral intensity of equilibrium radiation for the temperature range considered.

NOTATION

x, dimensionless coordinate; ξ_1, dimensionless frequency; λ, thermal conductivity, W/(m·K); k_ξ, spectral volumetric absorptivity, m^{-1}; τ_ξ, spectral optical thickness; D, material thickness, m; T, dimensionless temperature; Nr, radiation-conduction parameter; Q, dimensionless spectral volumetric density of the resultant radiation; $I_{\xi p}$, B_ξ, dimensionless spectral Planck and source functions, respectively; I_ξ, dimensionless spectral intensity of the radiation; E_n, $n = 1, 2, 3$, integro-exponential function of n-th order; ε_i^*, $i = 1, 2$, external integral emissivity of the i-th surface; $\varepsilon_{i\xi}$, $\rho_{i\xi}$, $i = 1, 2$, spectral emissivity and reflectivity of the i-th surface.

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